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Wave-induced oscillation in harbor of arbitrary shape with arbitrary reflection coefficient in uneven sea bed

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ABSTRACT: A numerical model to analyse wave induced oscillation in the harbor of arbitrary shape with quay or breakwater of arbitrary reflection coefficient in uneven sea bed, by means of Boundary Element Method (BEM) with be presented.

In order to examine the numerical model, the results of oscillation of a rectangular basin presented by Ippen & Goda, and by the present method are shown, good aggrements are obtained.

1 INTRODUCTION

The water surface oscillation problem is a very important factor for designing and planning in harbor engineering. When the oscillation occured in harbors the water surface will oscillate seriously, it will affect the efficiency of parking and loading of ships, even the safety of the harbor sometimes.

There were many studies concerned about the harbour oscillation problem, such as, Miles and Munk used Green function to analyse harbor oscillation with the radiation effects that expend from harbour entrance to offshore and the phenomenon of harbor paradox was found, Ippen and Goda used Fourier Transformation Method to analyse a rectangular basin, the experments were done also, good agreements were obtained. Berkhoff used mild-slope equation to solve wave diffraction and refraction problem in uneven sea. Lee used Weber's solution to solve Helmhotitz equation to analyse harbor oscillation of arbitrary shape in constant water depth. Chou used three dimensional BEM to analyse harbor oscillation of arbitrary shape with rigid quays. Chou developed a numerical model by means of two dimensional BEM to analyse harbor oscillation of arbitrary shape in constant water depth, but the quays have an arbitrary coefficient of reflection. Ou used Finite Element Method to solve this problem under the consideration of friction of sea beds.

In general, the harbors have a shape of arbitrary in uneven sea bed and the quays and breakwaters will have different coefficient of reflection that depending on its structures. However there was not such a numerical model that considered the whole factors mentioned above yet, in this paper a numerical model that considered the whole factors that affect the water surface oscillation in a harbor will be presented by 3 dimensional BEM, it is suitable to similate the water surface oscillations in a real harbor.

2 THEORETICAL ANALYSIS

As shown in Fig. 1, the fluid field is divided into two regions by the pseudo-boundary Γ_1: the open sea region with constant water depth (region I), and the region bounded by the pseudo-boundary Γ_1 and the limits of the harbor(region II). The pseudo-boundary Γ_1 is sufficient faraway from the harbor, therefore the scattering wave in region I induced by the harbor entrance and breakwater can be neglected.

Assuming the fluid in both regions is inviscid, incompressible and irrotational. When a regular incident wave with angular frequency $\sigma(=2\pi/T$, T is wave period), amplitude ζ_0 incidence from open sea with ω angle to x axis, the small amplitude wave motion in both regions exist velocity potential $\Phi(x,y,z;t)$ in the following form:

$$\Phi(x,y,z;t)=\frac{g\zeta_0}{\sigma}\phi(x,y,z)e^{-i\sigma t} \quad (1)$$

where g is acceleration of gravity, $\phi(x,y,z)$ must be satisfied by following equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \quad (2)$$

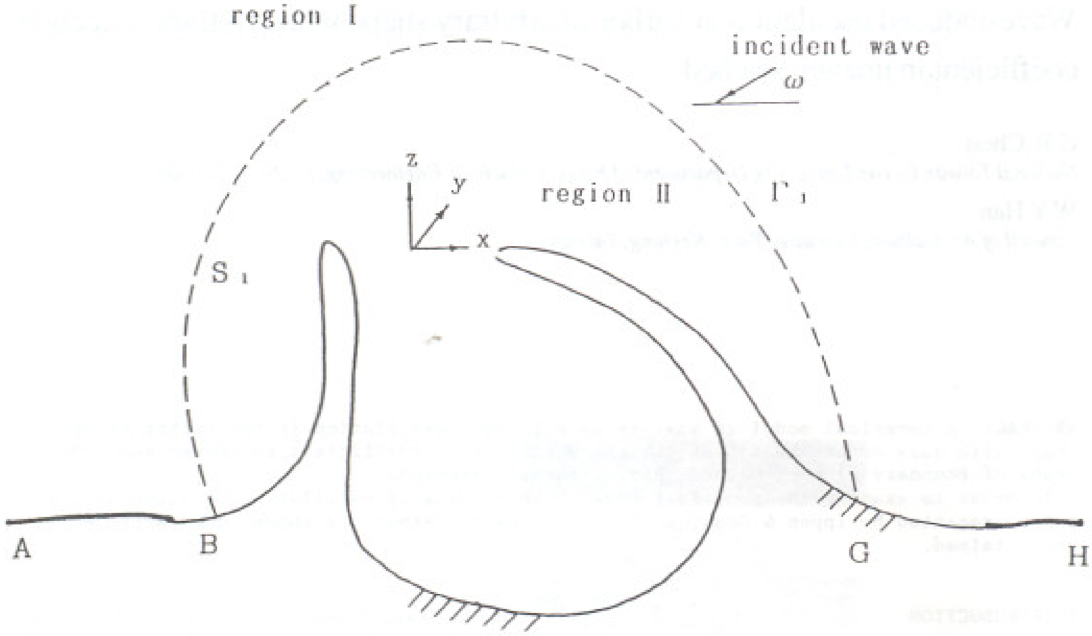


Fig.1 Definition sketch

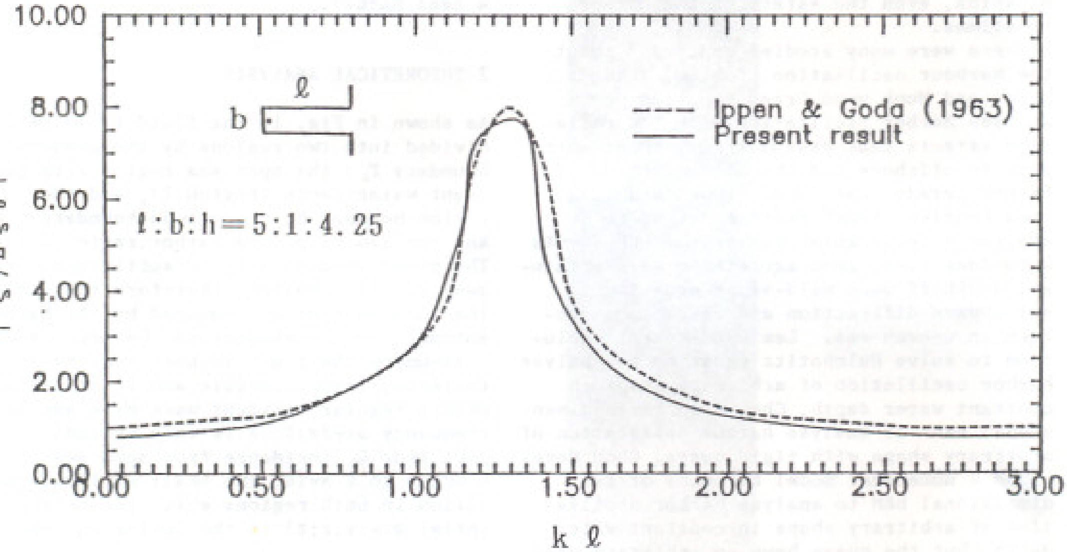


Fig.2 Respose curve at the center of the backwall of a full-open rectangular basin

(h = constant water depth of open sea)

2.1 Potential function in region I (constant water depth)

The potential function $\phi_1(x,y,z)$ in region I with constant water depth h due to the scattering wave induced by the existence of harbor entrance and breakwater are neglected, can be expressed as follows.

$$\phi_1(x,y,z) = [f^0(x,y)+f^*(x,y)]\frac{\cosh k(z+h)}{\cosh kh} \quad (3)$$

where k is the root of $\sigma^2 h/g = kh \tanh kh$, $f^0(x,y)$ is the potential function of the incident wave, and $f^*(x,y)$ is the potential function of the scattering wave.

If the incident wave is sinuoidal and incidence in an angle ω with x-axis, its water surface profiles will be expressed as follows

$$\zeta_i(x,y;t) = \zeta_0 \cos [k(x\cos\omega + y\sin\omega) + \sigma t], \quad (\pi \leqq \omega \leqq 0) \quad (4)$$

The potential function of incident wave $f^0(x,y)$ will be

$$f_0(x,y) = -i \cdot \exp[-ik(x\cos\omega + y\sin\omega)] \quad (5)$$

Substituting eq.3 into eq.2, we obtain the potential function of scattering wave $f^*(x,y)$ which satisfy the following Helmhotz equation.

$$\frac{\partial^2 f^*}{\partial x^2}+\frac{\partial^2 f^*}{\partial y^2}+k^2 f^* = 0 \quad (6)$$

The boundary of region I is limited by psuedo-boundary S_1, coastal line AB, GH and far field boundary. If we assume that psuedo-boundary S_1 is far away from harbor entrance, where the effects of scattering wave induced by the existence of harbor entrance and breakwaters can be neglected, the potential function of scattering wave $f^*(x,y)$ will be zero. At the far field boundary the radiation condition must be satisfied, $f^*(x,y)$ will be zero too. Therefore using Green function, the potential function $f^*(x,y)$ at any point inner region I can be calculated by following equation.

$$f^*(x,y) = \int_s \left[f^*(\xi,\eta)\frac{\partial}{\partial\nu}\left(-\frac{i}{4}H_0^{(1)}(kR)\right) - \left(-\frac{i}{4}H_0^{(1)}(kR)\right) \bar{f}^*(\xi,\eta)\right]dS \quad (7)$$

where $f^*(\xi,\eta)$ is the potential value at the boundary S_1, $\bar{f}^*(\xi,\eta) = \partial/\partial\nu\, f^*(\xi,\eta)$ is the boundary S_1, $H_0(kr)$ is Hankel function and R is a distance between (ξ,η) and (x,y).

For the purpose of numerical analysis, constant element will be used to discretize eq.5 on boundary S_1 by M elements. For the case c = 1/2, eq.5 can be expressed in matrix form as follows

$$\{F^*\} = [K^*]\ \{\bar{F}^*\} \quad (8)$$

$\{F^*\}$ and $\{\bar{F}^*\}$ are the potential function and its normal derivative on the boundary S_1 respectively, $[K^*]$ is a matrix of shape function.

2.2 Potential function in region II (uneven sea bed)

The boundary of region II is limited by psuedo-boundary Γ_1, water surface Γ_2, the quays or breakwater with arbitrary coefficientof reflection Γ_3 and impermeable uneven sea bed Γ_4. Using Green function, the potential function $\phi(x,y,z)$ inner the region II will be expressed as follows

$$c\,\phi(x,y,z) = \int \left[\bar{\phi}(\xi,\eta,\zeta)\left(\frac{1}{4\pi}\frac{1}{R}\right) - \phi(\xi,\eta,\zeta)\frac{\partial}{\partial\nu}\left(\frac{1}{4\pi}\frac{1}{R}\right)\right] dA \quad (9)$$

where $R = \sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$.

For the purpose of numerical calculation, constant element will be used to discretize eq.9 on each boundary $\Gamma_1 \sim \Gamma_4$ by $N_1 \sim N_4$ elements respectively. For the case c=1/2, eq.9 can be expressed in matrix form as follows

$$\{\phi\} = [K]\ \{\bar{\phi}\} \quad (10)$$

$\{\phi\}$ and $\{\bar{\phi}\}$ are the potential function and its normal derivative on the boundary $\Gamma_1 \sim \Gamma_4$ respectively, [K] is a matrix of shape function.

2.3 Boundary conditions on each boundary

Boundary condition on water surface:

Due to the air pressure is constant on water surface and the kinematic condition on the water surface, following boundary condition will be obtained

$$\bar{\phi} = \frac{\partial\phi}{\partial\nu} = \frac{\sigma^2}{g}\phi \qquad z=0 \quad (11)$$

Boundary condition on impermeable sea bed:

Assuming the sea bed is impermeable, we have

$$\bar{\phi} = \frac{\partial\phi}{\partial\nu} = 0 \quad (12)$$

Boundary condition on psuedo-boundary at open sea:

According to the mass flux and energy flux of fluid motion between region I and region II at the psuedo-boundary Γ_1 must be

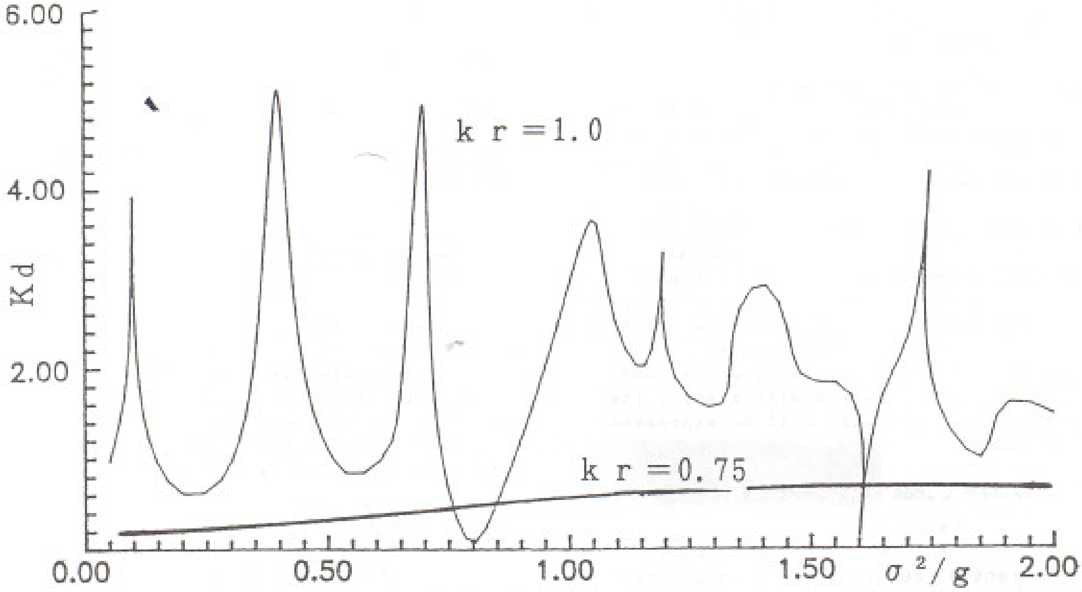


Fig. 3 Respose curve at the center of the backwall of a square basin with opening width $b=4h$, $\ell=10h$, $\omega=90^\circ$

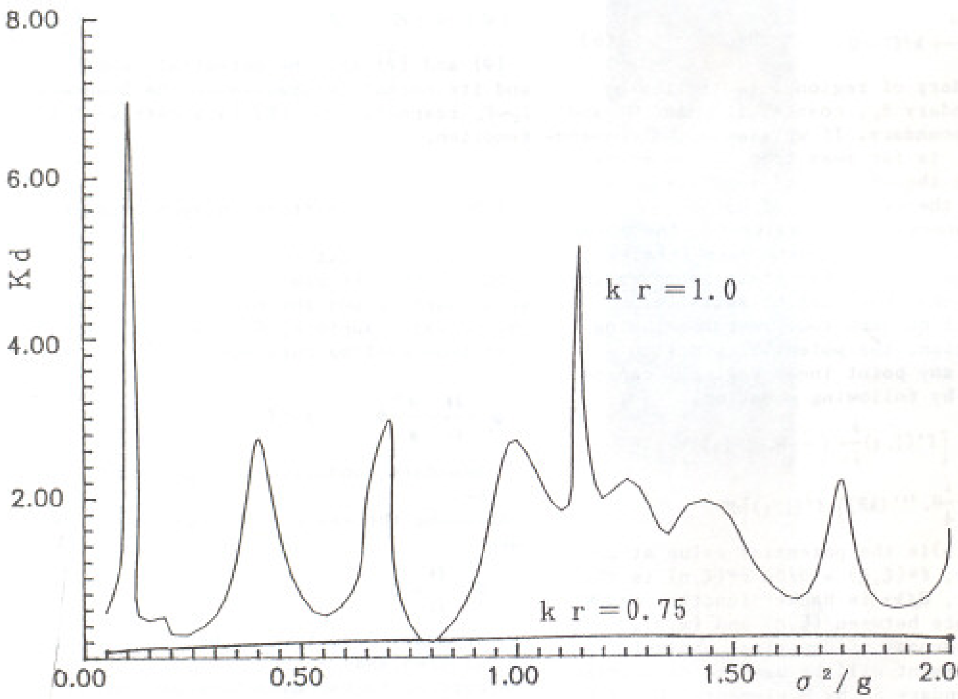


Fig. 4 Respose curve at the center of the backwall of a square basin with opening width $b=2h$, $\ell=10h$, $\omega=90^\circ$

conserved, we have

$$\bar{\phi}_o(\xi,\eta,\zeta)=\bar{\phi}(\xi,\eta,\zeta) \quad (13)$$

$$\phi_o(\xi,\eta,\zeta)=\phi(\xi,\eta,\zeta) \quad (14)$$

Substituting eq.3 into eq.13, multiply both sides with cosh k(z+h), and integrate from -h to 0, the following relation will be obtained.

$$\int_{-h}^{0}\bar{\phi}(\xi,\eta,\zeta)\cosh k(z+h)dz=\int_{-h}^{0}[\bar{f}_o(\xi,\eta)+\bar{f}^*(\xi,\eta)]\frac{\cosh^2 k(z+h)}{\cosh kh}dz$$

If we divide psuedo-boundary surface Γ_1 into n parts in the direction of water depth and m parts in horizontal direction, we have mxn elements on surface Γ_1. So we can discretilize above equation in a discrete form as follows

$$\bar{f}^*(\xi_i,\eta_i)=\frac{k}{N_o \sinh kh}\sum_{j=1}^{n}\bar{\phi}(\xi_i,\eta_i,\zeta_j)\cosh k(z_j+h)\Delta z_j-\bar{f}_o(\xi_i,\eta_i)$$

$$(i=1,2,\cdots,m) \quad (15)$$

where $N_0 = 0.5(1+2kh/\sinh 2kh)$.

Substituting eq.3 into eq.14, we obtain

$$\phi(\xi_i,\eta_i,\zeta_i)=[f_o(\xi_i,\eta_i)+f^*(\xi_i,\eta_i)]\frac{\cosh k(z_i+h)}{\cosh kh}$$

$$(i=1,2,\cdots,m\times n) \quad (16)$$

Substituting eqs.8,15 into eq.16, after some procedure of calculation, we obtain

$$\{\phi_1\}=[R]\{F_o-K^*\bar{F}_o\}+c[R][K^*][Q]\{\bar{\phi}_1\} \quad (17)$$

where $c=k/N_0\sinh kh$, subscript "1" means the functions on boundary Γ_1, [R] and [Q] are the matrix expressed as follows

$$R=\begin{bmatrix} \frac{\cosh k(z_{11}+h)}{\cosh kh} & & & & 0 \\ \vdots & & & & \\ \frac{\cosh k(z_{1n}+h)}{\cosh kh} & & & & \\ & & \ddots & & \\ 0 & & & & \frac{\cosh k(z_{m1}+h)}{\cosh kh} \\ & & & & \vdots \\ & & & & \frac{\cosh k(z_{mn}+h)}{\cosh kh} \end{bmatrix} \quad (18)$$

$$Q=\begin{bmatrix} \cosh k(z_{11}+h)\Delta z_1 \cdots \cosh k(z_{1n}+h)\Delta z_n & & & 0 \\ & \ddots & & \\ 0 & & \cosh k(z_{m1}+h)\Delta z_1 & \\ & & & \cdots\cosh k(z_{mn}+h)\Delta z_n \end{bmatrix} \quad (19)$$

Eq.17 expresses the relation between potential function $\phi(\xi,\eta,\zeta)$ and its derivative on the psuedo-boundary Γ_1.

Boundary condition on quay or breakwater with arbitrary coefficient of reflection:

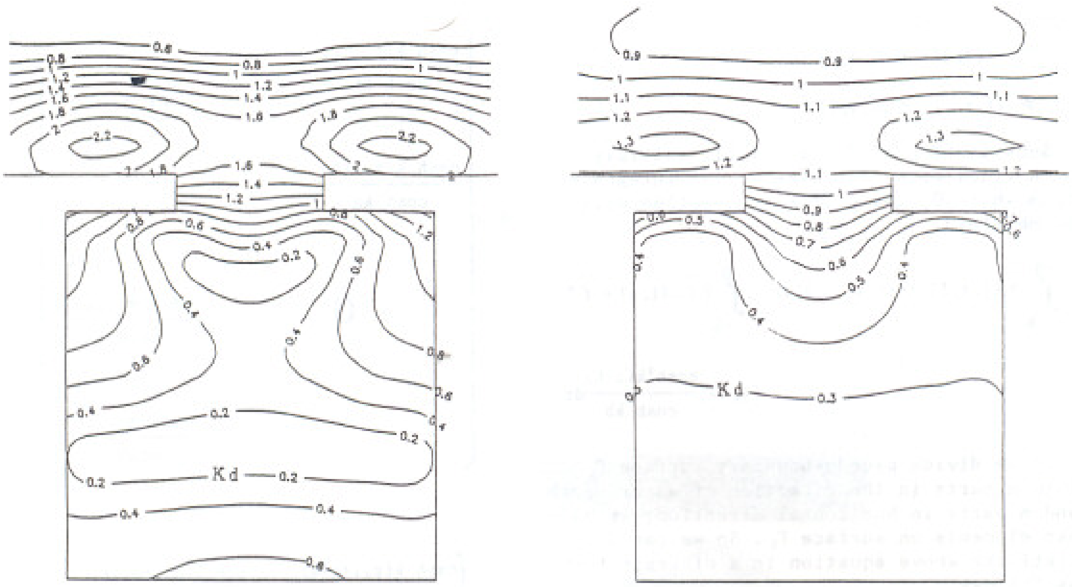
On quay or breakwater Γ_3 we assume coefficient of reflection is Kr, it means quay or breakwater has a coefficient of energy dissipation $\alpha(=\sqrt{1-Kr^2})$. Therefore boundary condition on Γ_3 can be expressed analogously as radiation condition as follows

$$\bar{\phi}(\xi,\eta,\zeta)=i\,k\,\alpha\,\phi(\xi,\eta,\zeta) \quad (20)$$

2.4 Establishment of simultaneous equations

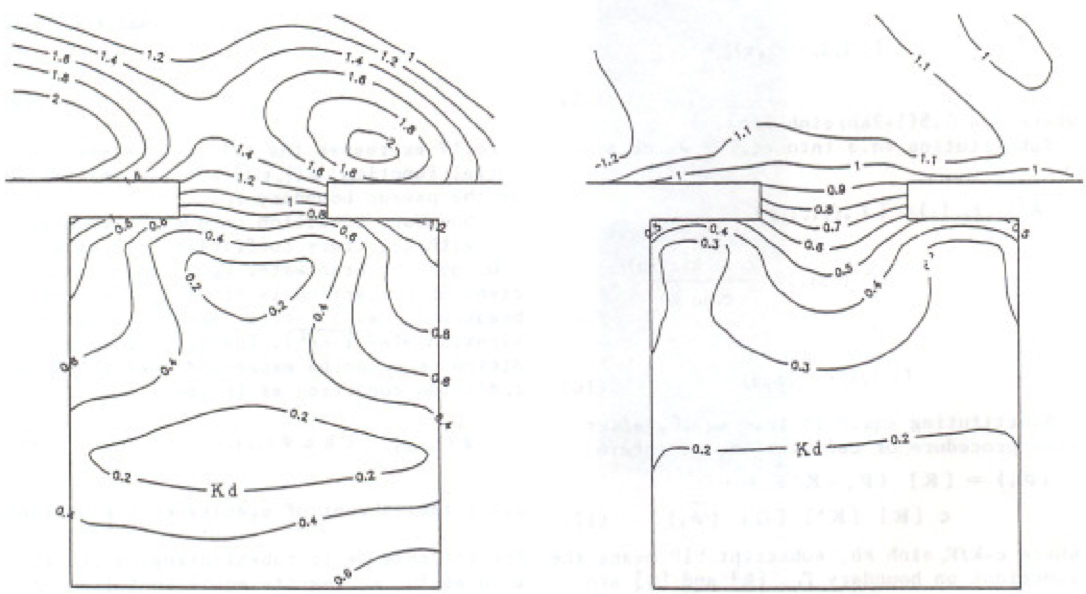
For convenience to substituting eqs.11-20 into eq.10, we rewrite eq.10 in following form

$$\begin{Bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{Bmatrix}=\begin{bmatrix}k_{11} & k_{12} & k_{13} & k_{14}\\ k_{21} & k_{22} & k_{23} & k_{24}\\ k_{31} & k_{32} & k_{33} & k_{34}\\ k_{41} & k_{42} & k_{43} & k_{44}\end{bmatrix}\begin{Bmatrix}\bar{\phi}_1\\ \bar{\phi}_2\\ \bar{\phi}_3\\ \bar{\phi}_4\end{Bmatrix} \quad (21)$$



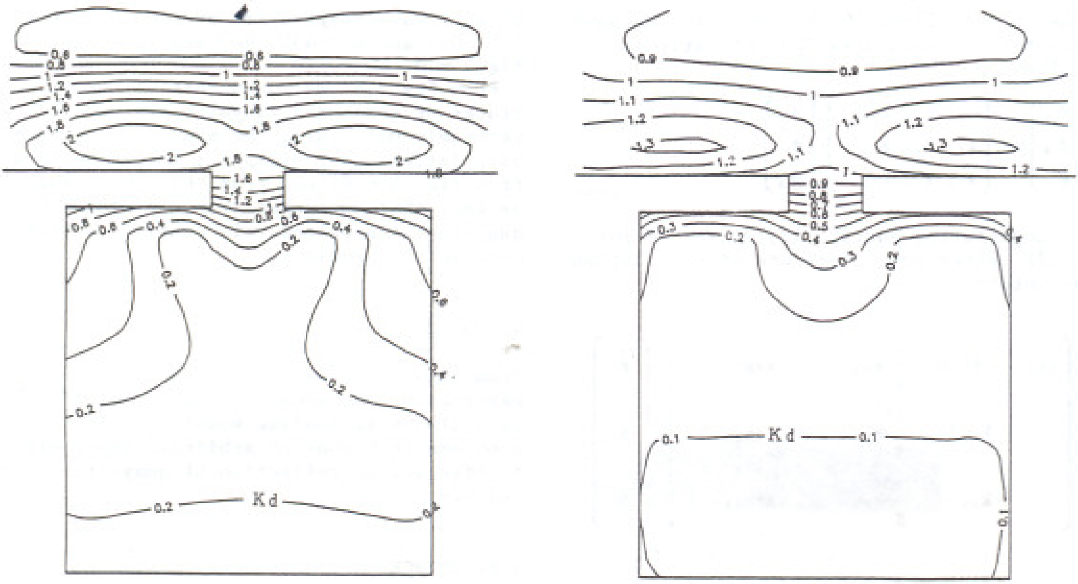
Kr = 1.0 Kr = 0.7

Fig. 5 Distribution of wave heights for b=4h, $\sigma^2/g=0.2$, $\omega=90°$



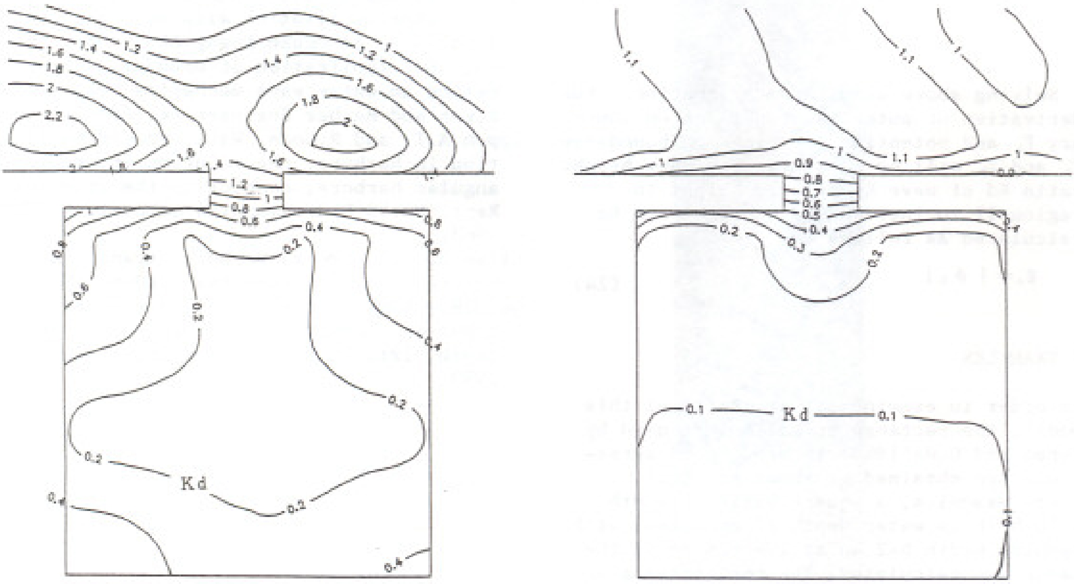
Kr = 1.0 Kr = 0.7

Fig. 6 Distribution of wave heights for b=4h, $\sigma^2/g=0.2$, $\omega=45°$



Kr = 1.0　　　　Kr = 0.7

Fig. 7 Distribution of wave heights for b= 2h, $\sigma^2/g = 0.2$, $\omega = 90°$



Kr = 1.0　　　　Kr = 0.7

Fig. 8 Distribution of wave heights for b= 2h, $\sigma^2/g = 0.2$, $\omega = 45°$

Due to eqs.12 and 20, we can solve unknown function on boundarys $\Gamma_1 \sim \Gamma_3$ firstly as follows

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \overline{\phi}_1 \\ \overline{\phi}_2 \\ \overline{\phi}_3 \end{bmatrix} \tag{22}$$

Substituting eqs.11, 17, 21 and 38 into eq.22, after some procedure of calculation we obtain

$$\begin{bmatrix} [k_{11}-cRk^{*}Q] & \frac{\sigma^2}{g}k_{12} & ik\sigma k_{13} \\ k_{21} & \frac{\sigma^2}{g}k_{22}-I & ik\sigma k_{23} \\ k_{31} & \frac{\sigma^2}{g}k_{32} & ik\sigma k_{33}-I \end{bmatrix} \begin{bmatrix} \overline{\phi}_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

$$= \begin{bmatrix} R[F_0-k^{*}\overline{F}_0] \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

Solving above simultaneous equations, the derivative of potential functions on boundary Γ_1 and potential functions on boundarys Γ_2 and Γ_3 will be obtained. The wave height ratio Kd of wave Kd of wave height in region II to incident wave height can be calculated as follows

$$K_d = |\phi_2| \tag{24}$$

3 EXAMPLES

In order to examine the exactness of this model, the rectangular basin that used by Ippen and Goda(1963) is used, good agreements are obtained as shown in Fig.2.

For examples, a square basin of width = 10 h (h is water depth of open sea) with opening width b=2,4h at the center of the basin are calculated. The coefficients of reflection of quay and breakwter Kr = 1.0, 0.75 are used. For the case of wave incidence perpendicular to coastal line, the time histories of water surface oscillations at the center of back-wall of the basin are shown in Fig.3, 4. The distributions of equi-wave heights for $\sigma^2 h/g = 0.2$, Kr= 1.0, 0.7 and $\omega = 45°$, $90°$ are shown in Fig.5 ~ 8.

From Fig.3,4, we found that if the structures of quay or breakwater are made by permeable materials, the phenomena of harbor oscillations will disapper, it can be known from Fig. 5 ~ 8 too. From this phenomena we know that using permable structures for quays or breakwaters will benefit to harbor oscillation problems.

4 CONCLUSION

From above procedure of calculation and examination, we know this numerical model is suitable to analyse water oscillation problems in harbor of arbitrary shape with coefficient of reflection of quays in uneven sea bed.

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