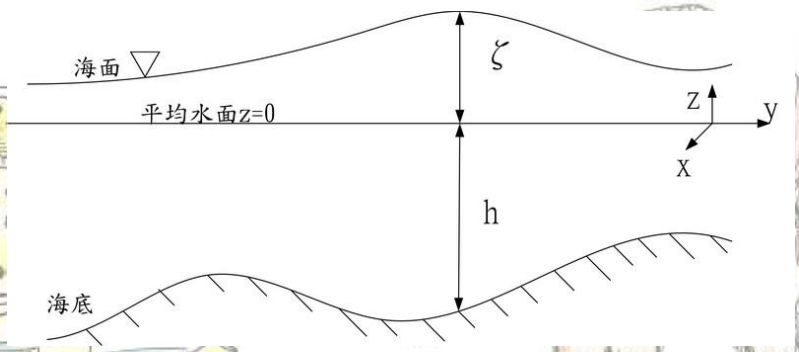
淺水長波連續方程式(Continuous equation in shallow water)



淺水海域內，在水平面內取直角座標系(x, y), z 軸垂直向上，潮流在 x, y 方向分量為 u, v。流體密度一定不可壓縮流的連續方程式如下

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

將上式對水深方向積分得

$$\int_{-h}^{\zeta}\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]dz=-\int_{-h}^{\zeta}\frac{\partial w}{\partial z}dz=-w|_{\zeta}+w|_{-h}$$

潮流屬長週期波運動，其運動特性為水粒子的水平速度只為 x，y 的函數，因此可將上式改寫成

$$\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]\int_{-h}^{\zeta}dz=-w|_{\zeta}+w|_{-h} \tag{2}$$

$$w|_{\zeta}=\frac{d\zeta}{dt}=\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y} \tag{3}$$

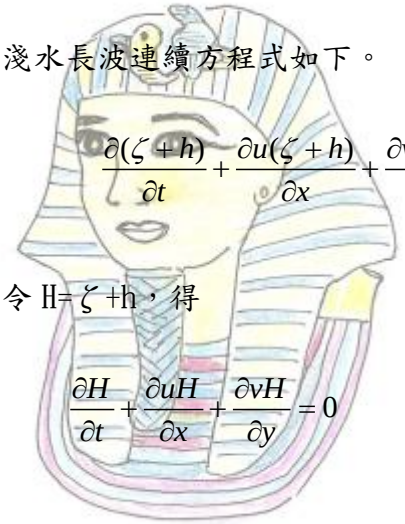
$$w|_{-h}=-\frac{dh}{dt}=-\frac{\partial h}{\partial t}-u\frac{\partial h}{\partial x}-v\frac{\partial h}{\partial y} \tag{4}$$

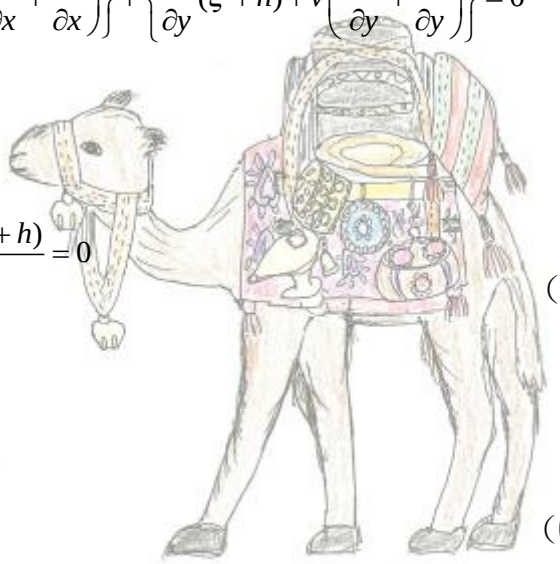
將(3)、(4)式代入(2)式得

$$\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right](\zeta+h)=-\frac{\partial\zeta}{\partial t}-\frac{\partial h}{\partial t}-u\left(\frac{\partial\zeta}{\partial x}+\frac{\partial h}{\partial x}\right)-v\left(\frac{\partial\zeta}{\partial y}+\frac{\partial h}{\partial y}\right)$$

即

$$\frac{\partial(\zeta+h)}{\partial t}+\left\{\frac{\partial u}{\partial x}(\zeta+h)+u\left(\frac{\partial \zeta}{\partial x}+\frac{\partial h}{\partial x}\right)\right\}+\left\{\frac{\partial v}{\partial y}(\zeta+h)+v\left(\frac{\partial \zeta}{\partial y}+\frac{\partial h}{\partial y}\right)\right\}=0$$





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得淺水長波連續方程式如下。

$$\frac{\partial(\zeta+h)}{\partial t}+\frac{\partial u(\zeta+h)}{\partial x}+\frac{\partial v(\zeta+h)}{\partial y}=0 \tag{5}$$

若令 H=ζ+h，得

$$\frac{\partial H}{\partial t}+\frac{\partial uH}{\partial x}+\frac{\partial vH}{\partial y}=0 \tag{6}$$

或

$$\frac{dH}{dt}+H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{7}$$

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若忽略非線性移流項得線性長波連續方程式如下。

$$\frac{\partial H}{\partial t}+H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{8}$$

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若海底為固定底床，因

$$\frac{\partial h}{\partial t}=0$$

得長波連續方程式如下。

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$$\frac{\partial \zeta}{\partial t}+\frac{\partial uH}{\partial x}+\frac{\partial vH}{\partial y}=0 \tag{9}$$

線性長波連續方程式則為

$$\frac{\partial \zeta}{\partial t}+H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \qquad (10)$$

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