能量守恆方程式(Energy conservation equation)

流體的比熱為 c，溫度為 T 時，領域內部熱能的變化率為

$$\int \frac{\partial}{\partial t}(\rho cT)d\Omega$$

從邊界流入熱能的流入率為

$$-\int\left(\rho cTv-k\frac{\partial T}{\partial n}\right)d\Gamma$$

兩者相等，得下列能量守恆法則

$$\int \frac{\partial}{\partial t}(\rho cT)d\Omega=-\int\left(\rho cTv-k\frac{\partial T}{\partial n}\right)d\Gamma$$

k 為熱傳導率。利用發散定理將上式右邊的邊界積分轉換成領域積分，得

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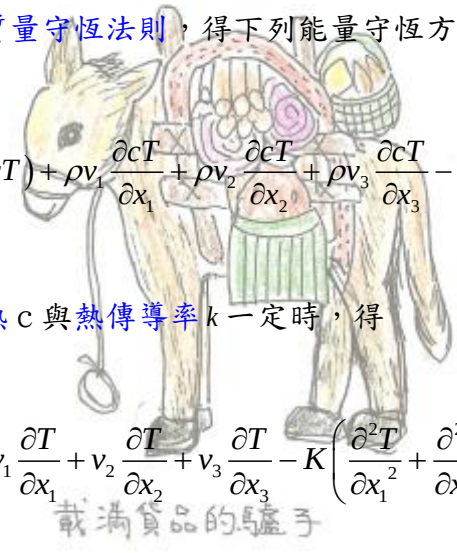
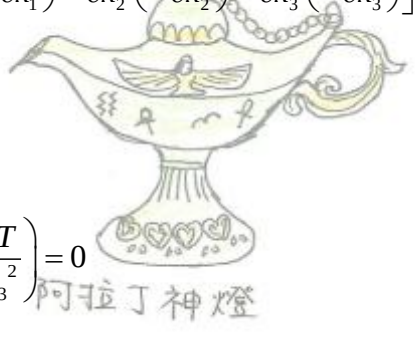
$$\int \frac{\partial}{\partial t}(\rho cT)d\Omega=-\int\left(\frac{\partial \rho cTv_1}{\partial x_1}+\frac{\partial \rho cTv_2}{\partial x_2}+\frac{\partial \rho cTv_3}{\partial x_3}\right)d\Omega+\int\left[\frac{\partial}{\partial x_1}\left(k\frac{\partial T}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(k\frac{\partial T}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(k\frac{\partial T}{\partial x_3}\right)\right]d\Omega$$

考量質量守恆法則，得下列能量守恆方程式。

$$\rho\frac{\partial}{\partial t}(cT)+\rho v_1\frac{\partial cT}{\partial x_1}+\rho v_2\frac{\partial cT}{\partial x_2}+\rho v_3\frac{\partial cT}{\partial x_3}-\left[\frac{\partial}{\partial x_1}\left(k\frac{\partial T}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(k\frac{\partial T}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(k\frac{\partial T}{\partial x_3}\right)\right]=0$$





當比熱 c 與熱傳導率 k 一定時，得

$$\frac{\partial T}{\partial t}+v_1\frac{\partial T}{\partial x_1}+v_2\frac{\partial T}{\partial x_2}+v_3\frac{\partial T}{\partial x_3}-K\left(\frac{\partial^2 T}{\partial {x_1}^2}+\frac{\partial^2 T}{\partial {x_2}^2}+\frac{\partial^2 T}{\partial {x_3}^2}\right)=0$$

溫度擴散率 K=k/(ρc)。

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