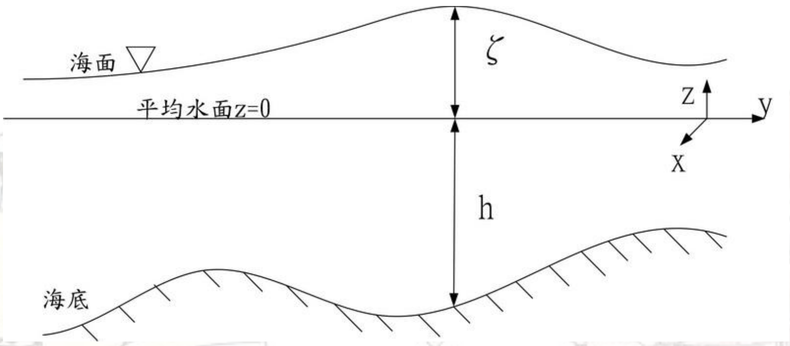
時間領域淺水長波邊界積分方程式



1. 支配方程式(詳參淺水長波支配及振動方程式)

淺水海域，在水平面內取直角座標系(x, y), z 軸垂直向上，潮汐流在 x, y 方向分量為 u、v，h 表示水深，水位上昇量為ζ時，淺水長波連續方程式為

$$\frac{\partial \zeta}{\partial t}+\frac{\partial u(\zeta+h)}{\partial x}+\frac{\partial v(\zeta+h)}{\partial y}=0 \qquad (1)$$

外力僅為重力時，x, y 方向的淺水長波運動方程式為

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial \zeta}{\partial x}=0 \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial \zeta}{\partial y}=0 \qquad (3)$$

將(1)式對時間微分，並將(2)式對 x 微分，(3)式對 y 微分後代入得

$$\nabla^2\zeta=\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}-\frac{1}{h}\left((1-g)\left(\nabla\zeta\bullet\nabla h\right)-\frac{\partial h}{\partial x}\frac{\partial u}{\partial t}-\frac{\partial h}{\partial y}\frac{\partial v}{\partial t}\right) \qquad (4)$$

上式為非線性淺水長波支配方程式。

將(2)及(3)式的移流項忽略，得下列線性淺水長波支配方程式

$$\nabla^2\zeta=\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}-\frac{1}{h}\left(\nabla\zeta\bullet\nabla h\right) \qquad (5)$$

2. 淺水長波支配方程式的邊界積分方程式化

2.1 非線性淺水長波邊界積分方程式

將(4)式乘以加權函數 G 並積分得

$$\int_{\Omega} G\nabla^2\zeta d\Omega = \int_{\Omega} G\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}d\Omega + \int_{\Omega} G\frac{1}{h}\left(\frac{\partial h}{\partial x}\frac{\partial u}{\partial t} + \frac{\partial h}{\partial y}\frac{\partial v}{\partial t} - (1-g)\left(\nabla\zeta \bullet \nabla h\right)\right)d\Omega \tag{6}$$

利用 Green 定理，上式改寫成

$$\begin{aligned}\int_{\Omega}\zeta\nabla^2 G d\Omega &= \int_{\Gamma} G\frac{\partial\zeta}{\partial n}d\Gamma - \int_{\Gamma}\zeta\frac{\partial G}{\partial n}d\Gamma + \int_{\Omega} G\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}d\Omega \\ &+ \int_{\Omega} G\frac{1}{h}\left(\frac{\partial h}{\partial x}\frac{\partial u}{\partial t} + \frac{\partial h}{\partial y}\frac{\partial v}{\partial t} - (1-g)\left(\nabla\zeta \bullet \nabla h\right)\right)d\Omega\end{aligned} \tag{7}$$

n 為邊界法線方向，上式左邊，加權函數 G 能滿足下式

$$\nabla^2 G = -\delta(Q-P)$$

其基本解為

$$G = \frac{1}{2\pi}\ell n\frac{1}{r}$$

即非線性潮汐流的邊界積分方程式可依下式表示

$$\gamma\zeta = \int_{\Gamma} G\frac{\partial\zeta}{\partial n}d\Gamma - \int_{\Gamma}\zeta\frac{\partial G}{\partial n}d\Gamma + \int_{\Omega} G\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}d\Omega + \int_{\Omega} G\frac{1}{h}\left(\frac{\partial h}{\partial x}\frac{\partial u}{\partial t} + \frac{\partial h}{\partial y}\frac{\partial v}{\partial t} - (1-g)\left(\nabla\zeta \bullet \nabla h\right)\right)d\Omega \tag{8}$$

在邊界上 γ=0.5，在領域內 γ=1。

2.2 線性淺水長波邊界積分方程式

將(5)式乘以加權函數 G 並積分得

$$\int_{\Omega} G\nabla^2\zeta d\Omega = \int_{\Omega} G\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}d\Omega - \int_{\Omega} G\frac{1}{h}(\nabla\zeta \bullet \nabla h)d\Omega$$

同理 2.1，利用 Green 定理，得下列線性淺水長波邊界積分方程式

$$\gamma\zeta = \int_{\Gamma} G\frac{\partial\zeta}{\partial n}d\Gamma - \int_{\Gamma}\zeta\frac{\partial G}{\partial n}d\Gamma + \int_{\Omega} G\frac{1}{gh}\frac{\partial^2\zeta}{\partial t^2}d\Omega - \int_{\Omega} G\frac{1}{h}(\nabla\zeta \bullet \nabla h)d\Omega \tag{9}$$