能量守恆法則

流體比熱為 c、溫度為 T 時，領域內部熱能的變化率為

$$\int \frac{\partial}{\partial t}(\rho c T)\, d\Omega$$

從邊界流入熱能的流入率為

$$-\int \left(\rho c T v - k \frac{\partial T}{\partial n} \right) d\Gamma$$

兩者相等，得下列能量守恆法則

$$\int \frac{\partial}{\partial t}(\rho c T)\, d\Omega = -\int \left(\rho c T v - k \frac{\partial T}{\partial n} \right) d\Gamma$$

k 為熱傳導率。利用發散定理將上式右邊的邊界積分轉換成領域積分，可得

$$\int \frac{\partial}{\partial t}(\rho c T) d\Omega = -\int \left(\frac{\partial \rho c T v_1}{\partial x_1} + \frac{\partial \rho c T v_2}{\partial x_2} \right) d\Omega + \int \left[\frac{\partial}{\partial x_1}\left(k \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2}\left(k \frac{\partial T}{\partial x_2} \right) \right] d\Omega$$

考量質量守恆法則，得下列能量守恆方程式。

$$\rho \frac{\partial}{\partial t}(cT) + \rho v_1 \frac{\partial cT}{\partial x_1} + \rho v_2 \frac{\partial cT}{\partial x_2} - \left[\frac{\partial}{\partial x_1}\left(k \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2}\left(k \frac{\partial T}{\partial x_2} \right) \right] = 0$$

當比熱 c 與熱傳導率 k 一定時，得

$$\frac{\partial T}{\partial t} + v_1 \frac{\partial T}{\partial x_1} + v_2 \frac{\partial T}{\partial x_2} - K \left(\frac{\partial^2 T}{\partial x_1{}^2} + \frac{\partial^2 T}{\partial x_2{}^2} \right) = 0$$

溫度擴散率 K=k/(ρc)。

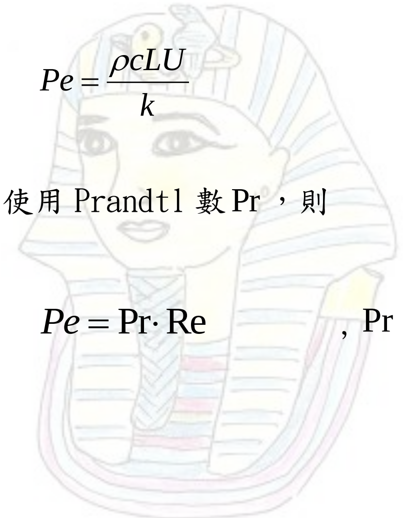
無因次化能量守恆方程式為

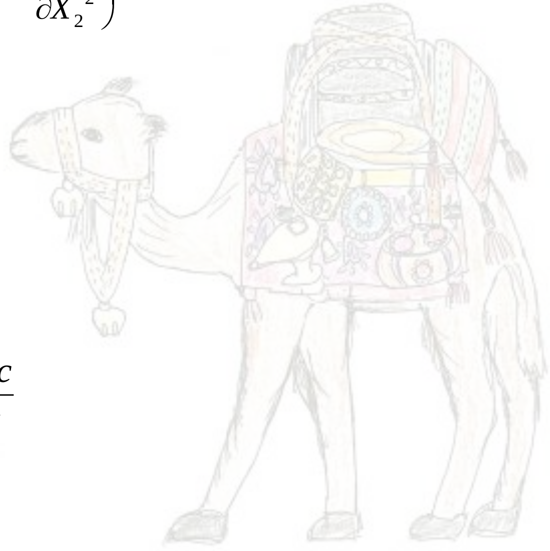
$$\frac{\partial \theta}{\partial \tau}+u_1\frac{\partial \theta}{\partial X_1}+u_2\frac{\partial \theta}{\partial X_2}+\frac{1}{Pe}\left(\frac{\partial^2 \theta}{\partial {X_1}^2}+\frac{\partial^2 \theta}{\partial {X_2}^2}\right)=0$$

$$Pe=\frac{\rho c L U}{k}$$

若使用 Prandtl 數 Pr，則

$$Pe=\Pr\cdot \mathrm{Re}\quad , \Pr=\frac{\mu c}{k}$$





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